
ASSIGNMENT-2

M.Sc.(Maths)-2nd Semester

Subject- Measure Theory and Integration

1. Let f be a bounded function on $[a,b]$ if f is R-integrable on $[a,b]$. Show that it is Lebesgue measurable and $R\int_a^b f(x)dx = L\int_a^b f(x)dx$.
2. Show that the function $f:[0,\infty)\to R$ defined by $f(x)=\begin{cases}\dfrac{\sin x}{x} & if \quad x\neq 0\\ 0 & if \quad x=0\end{cases}$ is not Lebesgue integrable over $[0,\infty)$.
3. Show that the theorem of Bounded convergence applies to $f_n(x)=\dfrac{nx}{1+n^2x^2},\ 0\le x\le 1.$
4. A measurable function f is integrable over E iff $|f|$ is integrable over E.
5. State Monotone Convergence theorem. Show that it need not hold for decreasing sequences of functions.
6. Give an example of a function which is not integrable in the sense of Lebesgue.